NATIONAL SENIOR CERTIFICATE EXAMINATION
SUPPLEMENTARY EXAMINATION MARCH 2016

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a) (1)
$$= \frac{(2x+13)(x-1)}{x-1}$$
$$= 2x+13$$

(2)
$$2x+13 = x^2$$
$$x^2 - 2x - 13 = 0$$
$$x = \frac{2 \pm \sqrt{56}}{2}$$
$$x = 1 \pm \sqrt{14}$$

(b) (1)
$$\frac{2^{9x}.2^{6x}}{2^{15x-2}}$$
$$= \frac{2^{15x}}{2^{15x-2}}$$
$$= 2^2$$

(2)
$$2^m = 2^2$$
$$m = 2$$

(c) (1)
$$x = -2$$
$$y = 1$$

(2)
$$0 = \frac{3}{x+2} + 1$$
$$0 = 3 + x + 2$$
$$x = -5$$
$$A(-5;\ 0)$$
New Equation
$$y = \frac{3}{x-3} + 1$$

QUESTION 2

(a) $p-3=9 \ \therefore p=12$
$25-12=q \ \therefore q=13$

(b) $2a=4$
$a=2$

$3(2)+b=9$
$b=3$

$2+3+c=3$
$c=-2$

$T_n=2n^2+3n-2$

QUESTION 3

(a) 10; 11; 12,1; …

(1) $T_8=10\ (1,1)^7 = 19{,}487171$ km

(2) $S_8=\dfrac{10[(1,1)^8-1]}{1,1-1} = 114{,}358881$ km

(b) (1) $a+4d=0$
$a+13d=-36$
$d=\dfrac{-36}{9}=-4$
$a+4(-4)=0$
$T_1=16$

(2) $d=-4$
$16+22(-4)+16+(23-p-1)(-4)=-96$
$-56-88+4p=-96$
$4p=48$
$p=12$

(c) (1) S_∞ exists because $r = \frac{2}{3}$ and $-1 < \frac{2}{3} < 1$

(2) $$S_\infty = \frac{-4}{1-\frac{2}{3}}$$
$$= -12$$

QUESTION 4

(a) $$301\,964 = \frac{x\left[1-\left(1+\frac{0,11}{12}\right)^{-120}\right]}{\frac{0,11}{12}}$$
$$x = R4\,159,55$$

(b) $$BO = \frac{4\,159,55\left[1-\left(1+\frac{0,11}{12}\right)^{-36}\right]}{\frac{0,11}{12}}$$
$$BO = R127\,053,07$$

QUESTION 5

(a) $$f'(x) = \lim_{h\to 0} \frac{(x+h)^2+\pi-(x^2+\pi)}{h}$$
$$= \lim_{h\to 0} \frac{x^2+2xh+h^2+\pi-x^2-\pi}{h}$$
$$= \lim_{h\to 0} \frac{h(2x+h)}{h}$$
$$= \lim_{h\to 0} 2x+h$$
$$f'(x) = 2x$$

(b) $$f(x) = ax + 5 + cx^{-1}$$
$$f^1(x) = a - cx^{-2} = \frac{ax^2 - c}{x^2}$$
$$\therefore a = 1 \text{ and } c = 3$$

QUESTION 6

(a) $\sqrt{x-2} = 4 - x$

$x - 2 = 16 - 8x + x^2$

$0 = x^2 - 9x + 18$

$(x-6)(x-3) = 0$

$x = 6 \text{ or } x = 3$

Check:

$\sqrt{6-2} + 6 = 8 \neq 4$

$\sqrt{3-2} + 3 = 4$

$\therefore x = 3$

(b) $x^2 - 4x - 4 + x < 0$

$x^2 - 3x - 4 < 0$

$(x-4)(x+1) < 0$

$+ \qquad\qquad - \qquad\qquad +$

$- - - - - - - - - - - - - - - - - - -$

$\qquad -1 \qquad\qquad 4$

$-1 < x < 4 \qquad or \qquad x \in (-1;\ 4)$

(c) (1) C

(2) D

(3) D

(4) B

SECTION B

QUESTION 7

(a) $10\,147\left(1+\frac{0{,}15}{4}\right)^{12}(1+0{,}15)^{9}+23\,786\,(1+0{,}15)^{4}$

$= R97\,125{,}19$

(b) $A=\frac{2\,172\left((1+0{,}15)^{12}-1\right)}{0{,}15}$

$A = R62\,991{,}62$

(c) Young ladies investment n years after 12 years

$= 62\,991{,}62\,(1+0{,}15)^{n}+\frac{2\,172\,[(1+0{,}15)^{n}-1]}{0{,}15} = 77\,471{,}12\,(1{,}15)^{n}-14\,480$

Young man's investment n years after 12 years

$= 97\,125{,}15\,(1{,}15)^{n}$

Now, $77\,471{,}12\,(1{,}15)^{n}-14\,480$
$< 77\,471{,}12\,(1{,}15)^{n}$
$< 97\,125{,}15\,(1{,}15)^{n}$
$\therefore$ true

QUESTION 8

(a) (1) $(4\times3\times2\times1)\times2 = 48$ *ways*

(2) $4\times3\times1+4\times3\times1 = 24$

(3) $5\times4\times3\times2+5\times4\times3\times2\times1$
$= 240$ *numbers*

(b) (1) $\frac{1}{6}$

(2) $\frac{7}{36}$

	1	2	3	4	5	6
1	x	x	x	x	x	x
2	x	x	x	x	x	x
3	x	x	x	x	x	y
4	x	x	x	x	y	y
5	x	x	x	x	x	x
6	x	x	y	y	y	y

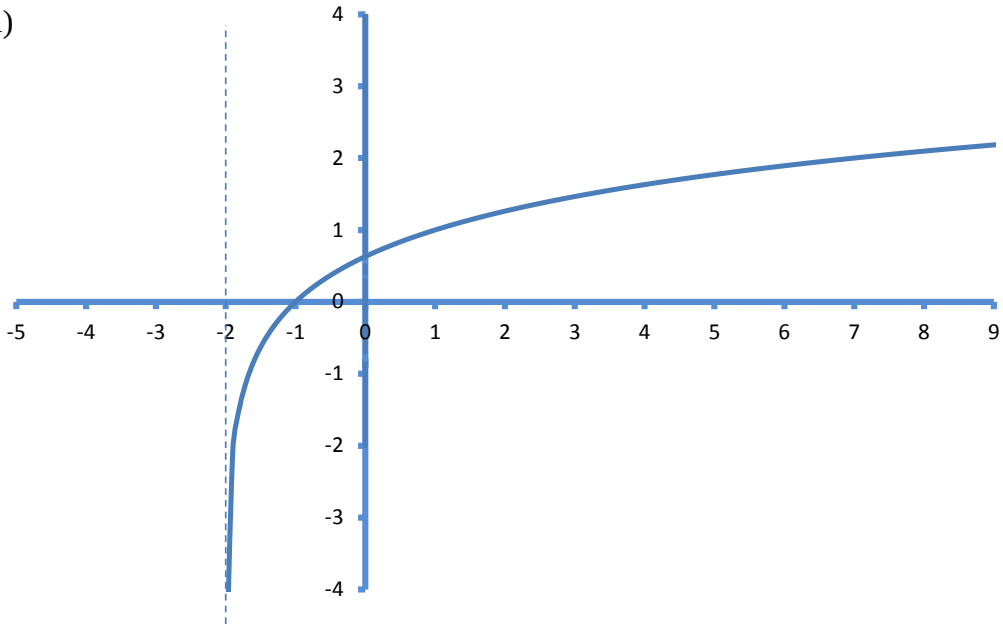
QUESTION 9

(a) (1) $x = 3^y - 2$

$x + 2 = 3^y$

$y = \log_3(x + 2)$

(2) (i)



(ii) $x \in (-1;\infty)$ or $x > -1$

(3) $g(x) = \dfrac{x-2-4}{x-2}$

$g(x) = \dfrac{x-2}{x-2} - \dfrac{4}{x-2}$

$g(x) = \dfrac{-4}{x-2} + 1$

$\therefore$ point of intersection of asymptotes is (2;1)

$\therefore h(x) = 3^x - 8$

(b) (1) $0 = -x^2 - 4x$

$x(x+4) = 0$

$x = 0$ *or* $x = -4$

$-2x - 4 = 0$

$x = -2$

$y = -(-2)^2 - 4(-2)$

$y = 4$

$\Delta AOD = \dfrac{1}{2} \times 4 \times 4$

$= 8$ *square units*

(2) $k < -1$

(3) $-(x-p)^2 - 4(x-p)$
$= -(x^2 - 2px + p^2) - 4x + 4p$
$= -x^2 + 2px - p^2 - 4x + 4p$
$B(0;-5)$
$\therefore -p^2 + 4p = -5$
$\therefore p^2 - 4p - 5 = 0 \quad \therefore p = -1 \quad or \quad p = 5$
$\therefore p < -1$

QUESTION 10

(a) $g'(x) = 3x^2 - 4x + k$
$3(3)^2 - 4(3) + k = 4$
$k = -11$

$p = (3)^3 - 2(3)^2 - 11(3) + 12$
$p = 27 - 18 - 33 + 12$
$p = -12$

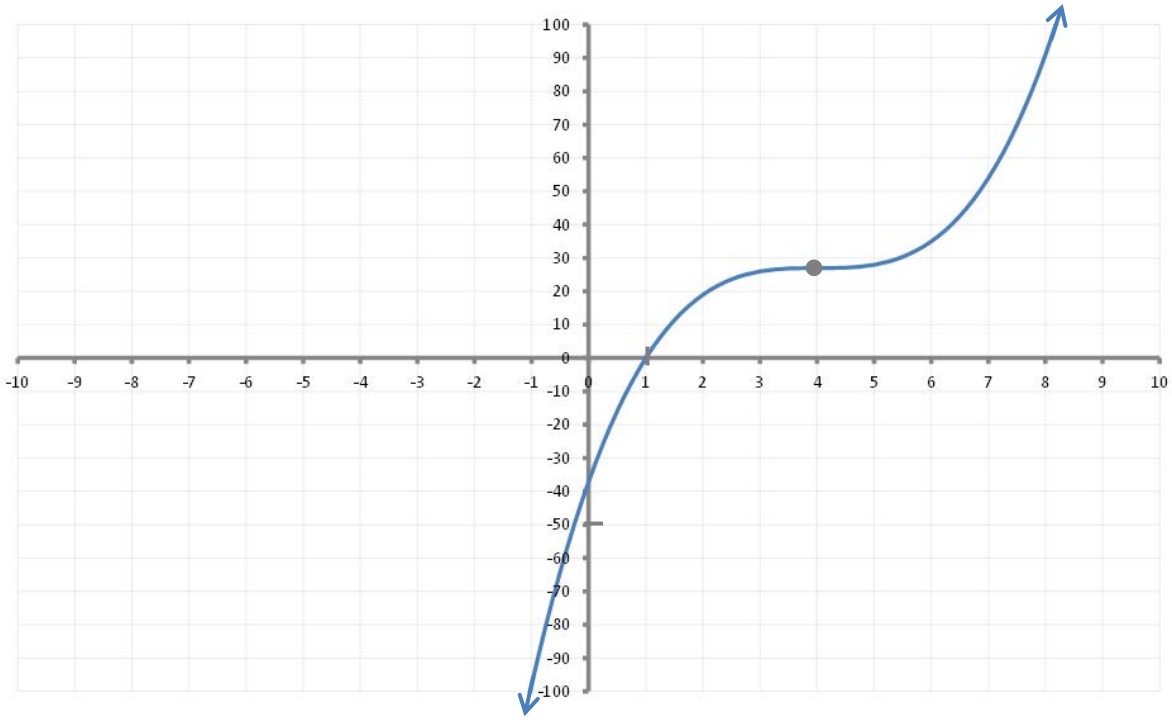
$-12 = 4(3) + t$
$t = -24$

$x^3 - 2x^2 - 11x + 12 = 4x - 24$
$x^3 - 2x^2 - 15x + 36 = 0$ or $(x-3)^2(x+4) = 0$
$(x-3)(x^2 + x - 12) = 0$ since 3 is a repeated root
$(x-3)(x-3)(x+4) = 0$

$y = 4(-4) - 24$
$y = -40$

$A(-4; -40)$

(b)



QUESTION 11

(a) $V_{outer} = \frac{2}{3}\pi(r+2)^3$

$= \frac{2}{3}\pi(r^3+6r^2+12r+8)$

$= \frac{2}{3}\pi r^3 + 4\pi r^2 + 8\pi r + \frac{16\pi}{3}$

$V_{inner} = \frac{2}{3}\pi r^3$

$\therefore$ volume of rubber

$= 4\pi r^2 + 8\pi r + \frac{16\pi}{3}$

(b) $\frac{dv}{dr} = 8\pi r + 8\pi$

$\therefore 8\pi p + 8\pi = \frac{88\pi}{3}$

$\therefore p + 1 = \frac{11}{3}$

$p = \frac{8}{3}\ cm$

Total: 150 marks