NATIONAL SENIOR CERTIFICATE EXAMINATION
SUPPLEMENTARY EXAMINATION – MARCH 2016

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours 150 marks

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Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13
18	10	9	18	12	9	13	11	10	10	10	14	6

TOTAL	/150

SECTION A

QUESTION 1

(a) $\text{AB} = \sqrt{(6-1)^2 + (-1+3)^2}$

$= \sqrt{29} \approx 5{,}4$

(b) $\text{M} = \left(\dfrac{-3+5}{2}; \dfrac{1+1}{2}\right)$

$\text{M} = (1; 1)$

(c) $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m_{AB} = \dfrac{5}{2}$

$m_{AD} = \dfrac{6-1}{-1-5} = -\dfrac{5}{6}$

(d) $\tan\theta = m_{AB} = \dfrac{5}{2}$

$\theta \approx 68°$

$\tan\,(180° - \alpha) = m_{AD} = -\dfrac{5}{6}$

$180° - \alpha = 140°$

$\alpha = 140°$

$\hat{\text{A}} \approx 180° - (68° + 40°)$

$\approx 72°$

(e) The diagonals of a parallelogram bisect each other

$\therefore$ M = Midpoint of AE

$(1; 1) = \left(\dfrac{-1 + x_{\text{E}}}{2}; \dfrac{6 + y_{\text{E}}}{2}\right)$

$-1 + x_{\text{E}} = 2,\ x_{\text{E}} = 3$

$6 + y_{\text{E}} = 2,\ y_{\text{E}} = -4$

E = (3; –4)

(f) $m_{AC} = \dfrac{6+4}{1+5} = \dfrac{5}{2} = m_{AB}$

$\therefore$ A, B and C lie on a straight line.

QUESTION 2

(a) (1) $(\text{max}) - (\text{min})$
$\text{Range} = 98 - 10$
$\text{Range} = 88$

(2) Median $=52$

(3) $\text{IQR} = Q_3 - Q_1$
$\text{IQR} = 70 - 44$
$\text{IQR} = 26$

(b) Science, higher median, higher minimum.

(c) Mathematics, Range is 88% and the IQR is 30 compared to Science with a Range of 58 and IQR of 26.

(d) 25% or 0,25

QUESTION 3

(a) $y = a + bx$
$y = 51{,}6478 + 0{,}7776x$

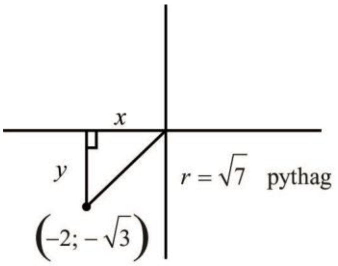
(b) $y = a + bx$
$y = 51{,}6478 + 0{,}7776\ (100)$
$y = 129$

(c) This prediction is not reliable as the maximum that could be obtained is 100.

(d) r = 0,8 which indicates a very strong positive correlation.

QUESTION 4

(a) (1)



$\sin\theta = \dfrac{-\sqrt{3}}{\sqrt{7}}$

$\cos\theta = \dfrac{-2}{\sqrt{7}}$

(2) $\sin 2\theta$

$= 2\sin\theta\cos\theta$

$= 2\left(\dfrac{-\sqrt{3}}{\sqrt{7}}\right)\left(\dfrac{-2}{\sqrt{7}}\right)$

$= \dfrac{4\sqrt{3}}{7}$

(3) $\cos^2(90° + \theta)$

$= (-\sin\theta)^2$

$= \sin^2\theta$

$= \left(\dfrac{-\sqrt{3}}{\sqrt{7}}\right)^2$

$= \dfrac{3}{7}$

(b) $\dfrac{\tan(180° - \theta).\cos(-\theta).\sin 390°}{\cos 300°.\sin\theta - \cos 450°}$

$= \dfrac{(-\tan\theta)(\cos\theta).\sin 390°}{\cos 300°.\sin\theta - \cos 450°}$

$= \dfrac{(-\tan\theta)(\cos\theta).\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)\sin\theta - 0}$

$= \dfrac{\left(\dfrac{-\sin\theta}{\cos\theta}\right)(\cos\theta)\left(\cancel{\frac{1}{2}}\right)}{\left(\cancel{\frac{1}{2}}\right)\sin\theta} = -1$

(c) (1) $\cos(A - 45°)$

$= \cos A\cdot\cos 45° + \sin A\cdot\sin 45°$

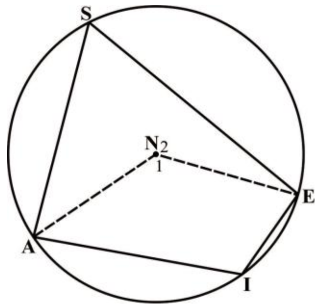
$= \dfrac{\sqrt{2}}{2}\cos A + \dfrac{\sqrt{2}}{2}\sin A$

$= \dfrac{\sqrt{2}}{2}(\cos A + \sin A)$

$\therefore \cos(A - 45°) = \dfrac{\sqrt{2}}{2}k$

(2) $1 + \sin 2A$
$= \sin^2 A + 2\sin A\cos A + \cos^2 A$
$= (\cos A + \sin A)^2$
$= k^2$

QUESTION 5



(a) $\hat{N}_1 = 2x$ *< at centre*

$\therefore\ \hat{N}_2 = 360° - 2x$ *angles around a point*

$\therefore\ \hat{I} = 180° - x$ *angle at centre*

$\therefore\ \hat{S} + \hat{I} = x + \hat{I} = 180°$

(b) $\hat{I}_1 = x\ ;\ \hat{I}_2 = y$

$\hat{K}_1 = x\ ;\ (GI = GK)$

$\hat{K}_2 = y\quad (JI = JK)$

$180° = \hat{I}_1 + \hat{I}_2 + \hat{K}_1 + \hat{K}_2$ *opposite angles of a cyclic quadrilateral*

$180° = x + y + x + y$

$180° = 2(x + y)$

$\therefore\ x + y = 90°$

$\therefore$ GJ is a diameter

GJ subtends $\angle$ of 90° at circumference

Alternate:

Since GI = GK, G lies on the perpendicular bisector of IK.
Since IJ = JK, J lies on the perpendicular bisector of IK.
$\therefore$ GJ is the perpendicular bisector of IK.
Hence the centre lies on GJ, proving GJ is a diameter of the circle.

QUESTION 6

$\text{O}\hat{\text{L}}\text{M} = 90°$ Tan $\perp$ Radius

$\hat{\text{O}}_2 = 64°$ Int. $\angle$s of ΔLOM

$\hat{\text{P}} = \hat{\text{L}}_2$ OP = OL

$\hat{\text{P}} + \hat{\text{L}}_2 = 64°$ ext. $\angle$ of Δ

$\therefore \hat{\text{L}}_2 = 32°$

$\hat{\text{L}}_5 = \hat{\text{P}}$; tan / chord theorem

$\hat{\text{P}} = \hat{\text{L}}_2 = 32°$

$\therefore \hat{\text{L}}_5 = 32°$

$\text{R}\hat{\text{L}}\text{O} = 90°$ tan $\perp$ radius

$\therefore \hat{\text{L}}_1 = 90° - 32° = 58°$

$\hat{\text{Q}}_1 = \hat{\text{L}}_1 = 58°$ tan-chord theorem

$\hat{\text{N}} = 32°$ tan-chord theorem

or $\angle$ in same segment

or $\angle$ at centre

NB:

Mark allocation

$\hat{\text{O}}_2$ (2)

$\hat{\text{L}}_2$ (3)

$\hat{\text{L}}_5$ (2)

$\hat{\text{Q}}_1$ (1)

$\hat{\text{N}}$ (1)

SECTION B

QUESTION 7

(a) (1) Sub $x = \frac{7}{5}$ and $y = -\frac{9}{5}$

$\left(\frac{7}{5}\right)^2 + \left(-\frac{9}{5}\right)^2 - 4\left(\frac{7}{5}\right) + 2\left(-\frac{9}{5}\right) + 4$

$= 0$

$\therefore \left(\frac{7}{5}\,;-\frac{9}{5}\right)$ is a point on circle

(2) Eq. of tangent: $y = mx + c$ at point $\left(\frac{7}{5};\frac{-9}{5}\right)$

$x^2 - 4x + y^2 + 2y = -4$

$(x-2)^2 + (y+1)^2 = -4 + 4 + 1$

$(x-2)^2 + (y+1)^2 = 1$

$\therefore$ Centre $(2;-1)$

$$m_{normal} = \frac{\frac{-9}{5}+1}{\frac{7}{5}-2}$$

$m_{normal} = \frac{4}{3}$ $\therefore$ $m_{\tan} = \frac{-3}{4}$

$y = \frac{-3}{4}x + c$ sub point $\left(\frac{7}{5};\frac{-9}{5}\right)$

$c = \frac{-3}{4}$

Eq. of tangent: $y = \frac{-3}{4}x - \frac{3}{4}$ or $4y = -3x - 3$

(b) Centre of circle = (4; –3)

Distance from (2; –1) to the centre

$= \sqrt{(4-2)^2 + (-3+1)^2}$

$= \sqrt{8}$

$\sqrt{8} > \sqrt{7}$

$\therefore$ Point lies outside

QUESTION 8

(a) In ΔOCA: Let OC = x

$\therefore$ EC = $(x+2)=\text{CA}$ radii

OA = 8 units

$(x+2)^2 = x^2+8^2$ pythag

$\therefore$ $x=15$

$\therefore$ C(0 ; -15) and radius = CP = 17

Sub. into: $(x-a)^2+(y-b)^2=r^2$

$(x-0)^2+(y+15)^2=17^2$

$x^2+y^2+30y-64=0$

(b) $\text{P}\left(\dfrac{x_1+x_2}{2};\dfrac{y_1+y_2}{2}\right)$

$\text{P}\left(\dfrac{-8+0}{2};\dfrac{0+0}{2}\right)$

$\text{P}(-4;0)$

$\therefore \text{Q}(-4;\text{y})$ sub $x=-4$ in circle equation

$(-4)^2+(y+15)^2=17^2$

$(y+15)^2=289-16$

$y+15\approx 16,5$

$y\approx 1,5$

Height of the pillar is 1,5 metres

QUESTION 9

(a) PT = $\frac{5}{8}(144)$

PT = 90 m

$T\hat{Q}P = 51°$ Ext. $\angle$ of ΔQPT

$$\frac{QT}{\sin 63°} = \frac{90}{\sin 51°}$$

QT $\approx$ 103 m

(b) In ΔQTS: TS = 54 m and QT = 103 m

$QS^2 = TS^2 + QT^2 - 2.TS.QT.\cos 114°$

$QS^2 = (54)^2 + (103)^2 - [2(54)(103)(\cos 114°)$

QS = $\sqrt{18049,53842}$

QS = 134, 3485706

$\tan Q\hat{S}R = \frac{70}{134,3485706}$

$Q\hat{S}R \approx 28°$ which is the angle of elevation of R from S

QUESTION 10

(a)

$$\frac{\sin 3\alpha}{\sin\alpha}$$
$$= \frac{\sin(2\alpha+\alpha)}{\sin\alpha}$$
$$= \frac{(\sin 2\alpha)(\cos\alpha)+(\cos 2\alpha)(\sin\alpha)}{\sin\alpha}$$
$$= \frac{(2\sin\alpha.\cos\alpha)(\cos\alpha)+(\cos^2\alpha-\sin^2\alpha)(\sin\alpha)}{\sin\alpha}$$
$$= \frac{\sin\alpha(2\cos\alpha)(\cos\alpha)+(\cos^2\alpha-\sin^2\alpha)(\sin\alpha)}{\sin\alpha}$$
$$= (2\cos\alpha)(\cos\alpha)+(\cos^2\alpha-\sin^2\alpha)$$
$$= 2\cos^2\alpha+\cos^2\alpha-\sin^2\alpha$$
$$= 3\cos^2\alpha-\sin^2\alpha$$
$$= 3(1\text{-}\sin^2\alpha)-\sin^2\alpha$$
$$= 3\text{-}3\sin^2\alpha-\sin^2\alpha$$
$$= 3\text{-}4\sin^2\alpha$$
$$= \text{RHS}$$

OR

$$\frac{\sin 3\alpha}{\sin\alpha}$$
$$= \frac{\sin 2\alpha\ \cos\alpha + \cos 2\alpha.\sin\alpha}{\sin\alpha}$$
$$= \frac{2\sin\alpha\ \cos^2\alpha + \left(1-2\sin^2\alpha\right).\sin\alpha}{\sin\alpha}$$
$$= 2\cos^2\alpha + 1 - 2\sin^2\alpha$$
$$= 2 - 2\sin^2\alpha + 1 - 2\sin^2\alpha$$
$$= 3 - 4\sin^2\alpha$$
$$= \text{RHS}$$

(b)

$$3-4\sin^2\alpha = 2$$
$$4\sin^2\alpha = 1$$
$$\sin\alpha = \frac{1}{2}\ \text{OR}\ -\frac{1}{2}$$
$$\alpha = 30^\circ + 360k;\quad k\in\mathbb{Z}$$

OR

$$\alpha = 150^\circ + 360k;\quad k\in\mathbb{Z}$$

QUESTION 11

(a) $S\hat{W}T = 90°$ $\angle$ in semi-circle

$V\hat{U}S = 90°$ given

$\therefore$ WVUT is a cyclic quad. Converse of Ext angle of cyclic quadrilateral

(b) $U\hat{W}T = \hat{S}$ tan-chord theorem

$U\hat{W}T = \hat{V}_1$ $\angle$s in same segment

$\therefore \hat{V}_1 = S$

$\therefore$ VU is a tangent ; converse of tan/chord theorem

QUESTION 12

(a) In ΔSWU

$\frac{ST}{18} = \frac{8}{12}$ *line parallel to one side of triangle*

$\therefore ST = \frac{8 \times 18}{12} = 12$

In ΔXUS

$\frac{XW}{20} = \frac{12}{18}$ *line parallel to one side of triangle*

$\therefore XW = \frac{20 \times 12}{18} = \frac{40}{3}$

(b) Determine Area ΔSXU:

$\frac{VT}{20} = \frac{12}{20}$ *($\Delta VUT /// \Delta WUS$)*

$\therefore VT = 12$

$(12)^2 = (12)^2 + (18)^2 - 2(12)(18)\cos\hat{U}$

$\cos\hat{U} = \frac{3}{4}$

$\sin\hat{U} = \sqrt{1 - \cos 2\hat{U}} = \frac{\sqrt{7}}{4}$

$\text{Area} = \frac{1}{2}\left(33\frac{1}{3}\right)(30)\left(\frac{\sqrt{7}}{4}\right)$

$= 125\sqrt{7}$

QUESTION 13

(a) OR = ON = p radii

$\therefore \text{O}\hat{\text{R}}\text{N} = \hat{\text{N}}_1 = \alpha$ $\angle$ opp = sides

$\therefore \text{R}\hat{\text{O}}\text{N} = 180° - 2\alpha$ int. $\angle$s of Δ

Using the Cosine Rule:

$(\text{RN})^2 = p^2 + p^2 - [2.p.p.\cos(180° - 2\alpha)]$

$(\text{RN})^2 = 2p^2 - 2p^2(-\cos 2\alpha)$

$(\text{RN})^2 = 2p^2(1 + \cos 2\alpha)$

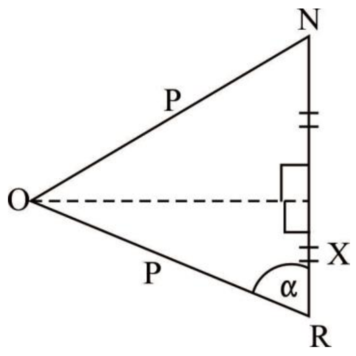
$(\text{RN})^2 = 2p^2[1 + (2\cos^2\alpha - 1)]$

$(\text{RN})^2 = 2p^2(2\cos^2\alpha)$

$(\text{RN})^2 = 4p^2\cos^2\alpha$

$\text{RN} = 2p\cos\alpha$

OR



$\cos\alpha = \dfrac{x}{p}$

$x = p\cos\alpha$

$\therefore \text{RN} = 2p\cos\alpha$

(b) RN = 20cos45°

= $10\sqrt{2}$

Perpendicular height = circumference of sphere = 25 m

Volume of pyramid = $\dfrac{1}{3}\left(10\sqrt{2} \times 10\sqrt{2}\right)(25)$

Volume of pyramid = 1 667 m^3

Volume of hemisphere = $\dfrac{2}{3} \times \pi \times 25^3$

= 3 2 724,92

$\therefore$ volume of water = 31 058 m^3

Total: 150 marks