NASIONALE SENIOR SERTIFIKAAT-EKSAMEN
AANVULLINGSEKSAMEN – MAART 2019

WISKUNDE: VRAESTEL I
NASIENRIGLYNE

Tyd: 3 uur | 150 punte

Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en hulp-eksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.

AFDELING A

VRAAG 1

(a) (1) $(37-x)-(x+5)=(x+13)-(37-x)$

$37-x-x-5=x+13-37+x$

$-4x=-56$

$x=14$

(2) $T_1=19, \quad T_2=23, \quad T_3=27$

$d=4$

$T_n=19+(n-1)(4)$

$T_n=4n+15$

(b) $S_3=\dfrac{a(r^n-1)}{r-1}; r\neq 1$

$91=\dfrac{a(3^3-1)}{3-1}$

Eerste term: $a=7$

Alternatief:

$a+3a+9a=91$

$a=7$

(c) $S_\infty=\dfrac{a}{1-r} \quad ;-1<r<1$

$\dfrac{375}{4}=\dfrac{a}{1-r}$

$\therefore 4a=375-375r$

$\therefore a=\dfrac{375}{4}(1-r)$ …vergelyking 1

$S_2=\dfrac{a(r^n-1)}{r-1} \quad ;r\neq 1$

$90=\dfrac{a(r^2-1)}{r-1}$

$90=\dfrac{a(r-1)(r+1)}{(r-1)}$

$90=a(r+1)$

$90=\dfrac{375}{4}(1-r)(r+1)$

$$90 = -\frac{375}{4}(r-1)(r+1)$$

$$90 = -\frac{375}{4}(r^2-1)$$

$$r^2 = \frac{1}{25}$$

$$\therefore r = \frac{1}{5} \quad \text{of} \quad r = -\frac{1}{5}$$

$$\text{en } \therefore a = \frac{375}{4}(1-r)$$

$$\therefore a = 75 \quad a = \frac{225}{2}$$

Alternatief:

$$S_\infty = \frac{a}{1-r} \quad ;-1 < r < 1$$

$$\frac{375}{4} = \frac{a}{1-r}$$

$$\therefore 4a = 375 - 375r$$

$$\therefore a = \frac{375}{4}(1-r) \text{ ...vergelyking 1}$$

$$S_2 = a + ar$$

$$S_2 = a(1+r)$$

$$90 = \frac{375}{4}(1-r)(1+r)$$

$$90 = -\frac{375}{4}(r^2-1)$$

$$r^2 = \frac{1}{25}$$

$$\therefore r = \frac{1}{5} \quad \text{of} \quad r = -\frac{1}{5}$$

$$\text{en } \therefore a = \frac{375}{4}(1-r)$$

$$\therefore a = 75 \quad a = \frac{225}{2}$$

(d) (1) Verskiltoets:

	32699		32896		33091		33284		33475
1ste verskil		197		195		193		191	
2de verskil			–2		–2		–2		

Konstante tweede verskil, dus kwadraties.

(2) $a+b+c=32699$

$3a+b=197$

$2a=-2$

$\therefore a=-1$

$\therefore b=200$

$\therefore c=32500$

$T_n=-n^2+200n+32500$

(3) $0=-2n+200$
$n=100$
Op die 100ste dag

Alternatief:
$-T_n=n^2-200n+\left(-100\right)^2-32500-\left(-100\right)^2$
$-T_n=(n-100)^2-42500$
$T_n=-(n-100)^2+42500$
Maksimum op die 100ste dag

VRAAG 2

(a)

$$f'(x) = \lim_{h \to 0} \frac{-(x+h)^2 + 2(x+h) - (-x^2 + 2x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{-x^2 - 2xh - h^2 + 2x + 2h + x^2 - 2x}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{h(-2x - h + 2)}{h}$$

$$f'(x) = \lim_{h \to 0}(-2x - h + 2)$$

$$f'(x) = -2x + 2$$

(b)

$g'(x) = 6x^2 + 6x$

$g'(-2) = 6(-2)^2 + 6(-2)$

$g'(-2) = 12$

Vir koördinaat van die punt van kontak:

$g(-2) = 2(-2)^3 + 3(-2)^2 + 1$

$g(-2) = -3$

Vervang: $(-2;-3)$ in $y = 12x + c$

$-3 = 12(-2) + c$

$c = 21$

Vergelyking van raaklyn: $y = 12x + 21$

(c) (1)

$y = \sqrt[3]{x^2} + 3x^2 - 4x$

$y = x^{\frac{2}{3}} + 3x^2 - 4x$

$\frac{dy}{dx} = \frac{2}{3}x^{-\frac{1}{3}} + 6x - 4$

$\therefore \frac{dy}{dx} = \frac{2}{3\sqrt[3]{x}} + 6x - 4$

(2) $y = (x+\pi)^{-1}(x^{-1}+\pi^{-1})$

$$y = \left(\frac{1}{x+\pi}\right)\left(\frac{1}{x}+\frac{1}{\pi}\right)$$

$$y = \left(\frac{1}{x+\pi}\right)\left(\frac{\pi+x}{\pi x}\right)$$

$$y = \left(\frac{1}{\pi x}\right)$$

$$y = \frac{1}{\pi}x^{-1}$$

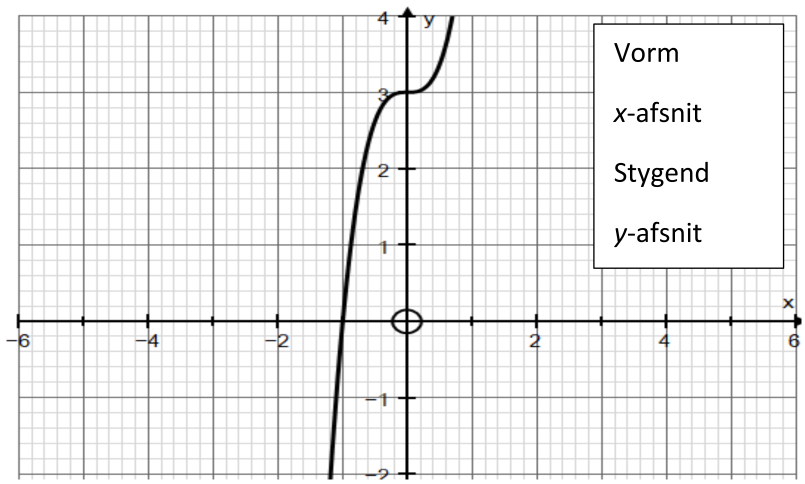
$$\frac{dy}{dx} = -\frac{1}{\pi}x^{-2}$$

$$\frac{dy}{dx} = -\frac{1}{\pi x^2}$$

VRAAG 3

(a) $f'(x) = 9x^2$, dus 'n positiewe gradiënt vir alle reële waardes van x.

(b)



(c) $(0;3)$

VRAAG 4

(a) $c = 3$

$f(-1) = a(-1)^2 + b(-1) + 3 = 0$

$a = b - 3$ vergelyking 1

$f(1) = a(1)^2 + b(1) + 3 = 2$

$a + b = -1$ vergelyking 2

$\therefore b - 3 + b = -1$

$\therefore b = 1$ en $a = -2$

(b) $f(x) = -2x^2 + x + 3$

$f'(x) = -4x + 1$

Vir dalende gradiënt: $f'(x) < 0$

$-4x + 1 < 0$

$x > \frac{1}{4}$

(c) $\therefore -3 = -2x^2 + x + 3$

$\therefore 0 = -2x^2 + x + 6$

$x = 2$; $x = -\frac{3}{2}$ (NVT)

$g(x) = \frac{d}{x} - 2$ … vervang $(2;-3)$

$-3 = \frac{d}{2} - 2$

$d = -2$

VRAAG 5

(a) $1116 = 558(1{,}08)^x$

$(1{,}08)^x = 2$

$x = \log_{1{,}08} 2$

$x \approx 9$ jaar

NB: Vir probeer en verbeter kan volpunte toegeken word.

(b) $x = 558(1{,}08)^y$

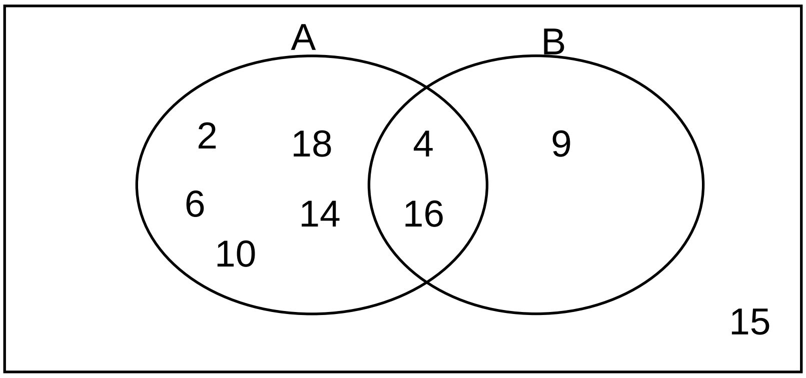
$\frac{x}{558} = (1{,}08)^y$

$y = \log_{1{,}08}\left(\frac{x}{558}\right)$

$\therefore f^{-1}(x) = \log_{1{,}08}\left(\frac{x}{558}\right)$

VRAAG 6

(a)



(b) $P(A \text{ of } B) = \frac{8}{9}$

(c) $P(A \text{ en } B) = \frac{2}{9}$

(d) $P(A' \text{ en } B') = \frac{1}{9}$

AFDELING B

VRAAG 7

(a) (1) $\text{P(almal sal slaag)} = \frac{4}{5} \times \frac{5}{7} \times \frac{2}{3}$

$\therefore \text{P(almal sal slaag)} = \frac{8}{21}$

(2) $\text{P(almal sal druip)} = \left(1 - \frac{4}{5}\right) \times \left(1 - \frac{5}{7}\right) \times \left(1 - \frac{2}{3}\right)$

$\text{P(almal sal druip)} = \frac{1}{5} \times \frac{2}{7} \times \frac{1}{3}$

$\text{P(almal sal druip)} = \frac{2}{105}$

(3) $\text{P(minstens een sal slaag)} = 1 - \text{P(almal sal druip)}$

$= 1 - \frac{2}{105}$

$= \frac{103}{105}$

(b)

(1) 26 letters in die alfabet

Getal moontlike wagwoorde: $26 \times 25 \times 24 \times 23 = 358\ 800$

Alternatief: $\frac{26!}{(26-4)!} = 358\ 800$

(2) $P\left(\text{ontsluit met eerste probeerslag}\right) = \frac{1}{358\ 800}$

(3) $P\left(\text{hy sal uitgesluit word}\right) = 1 - P\left(\text{hy sal nie uitgesluit word nie}\right)$

$= 1 - \left(\frac{1}{358\ 800} \times \frac{1}{358\ 799} \times \frac{1}{358\ 798}\right)$

≈ 1

VRAAG 8

(a) $A = P(1-i)^n$

$$A = 325000\left(1-\frac{7}{100}\right)^9$$

$A = 169\,133,602$

$$\%\text{ depresiasie } = \frac{169\,133,602}{325\,000}\times 100$$

$\%\text{ depresiasie } \approx 52\%$

(b) $$P = x\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$1\,825\,000 = x\left[\frac{1-\left(1+\frac{9,5}{100(12)}\right)^{-25\times 12}}{\left(\frac{9,5}{100(12)}\right)}\right]$$

$x = 15\,944,96406$

OPSIE 1 BEDRAG VERKRY:
= $(15\,944,96406\times 12\times 9)\times 75\%$
= $1\;291\,542,089$

OPSIE 2 BEDRAG VERKRY:

Saldo van die lening = A – F_v

$$A = 1\,825\,000\left(1+\frac{9,5}{100(12)}\right)^{9\times 12}$$

$A = 4\,276\,835,507$

$$F = x\left[\frac{(1+i)^n - 1}{i}\right]$$

$$F = 15\,944,96406\left[\frac{\left(1+\frac{9,5}{100(12)}\right)^{(9\times 12)} - 1}{\frac{9,5}{100(12)}}\right]$$

$F = 2\,705\,886,942$

Saldo van die lening = $4\,276\,835,507 - 2\,705\,886,942$
Saldo van die lening = $1\,570\,948,565$

Alternatief vir OPSIE 2:

$$\text{Saldo van die lening} = 15\,944{,}96406\left[\frac{1-\left(1+\frac{9{,}5}{1200}\right)^{-16\times 12}}{\frac{9{,}5}{1200}}\right]$$

$$\text{Saldo van die lening} = 1\,570\,948{,}565$$

Bedrag verkry $= 5\times\left(1\,825\,000 - 1\,570\,948{,}565\right)$

Bedrag verkry $= 1\,270\,257{,}175$

Dus sal OPSIE 1 die grootste bedrag oplewer.

VRAAG 9

(a) $2(x^2 - x) = a$

$x^2 - x = \frac{a}{2}$

$x^2 - x + \left(-\frac{1}{2}\right)^2 = \frac{a}{2} + \left(-\frac{1}{2}\right)^2$

$\left(x - \frac{1}{2}\right)^2 = \frac{a}{2} + \frac{1}{4}$

$x - \frac{1}{2} = \pm\sqrt{\frac{2a+1}{4}}$

$x = \frac{1}{2} \pm \frac{\sqrt{2a+1}}{2}$

$x = \frac{1 \pm \sqrt{2a+1}}{2}$

(b) (1) $\log_p\left[x.(x+p)\right] = 0$

$p^0 = x.(x+p)$

$1 = x^2 + xp$

$x^2 + xp - 1 = 0$

$x = \frac{-p \pm \sqrt{p^2+4}}{2}$

(2) $(x - p + 3)^2 = 4$

$x - p + 3 = \pm 2$

$x - p + 3 = 2$ of $x - p + 3 = -2$

$x = p - 1$ of $x = p - 5$

(c) (1) $\sqrt{x+4} = \dfrac{4}{\sqrt{x-2}}$

$\left(\sqrt{x+4}\right)\left(\sqrt{x-2}\right) = 4$

$\sqrt{x^2+2x-8} = 4$

$x^2+2x-24=0$

$x=4$ of $x=-6$ (NVT)

(2) $\left(2^x\right)^4 - 8.2^x = 0$ Laat $2^x = k$

$k^4-8k=0$

$k(k^3-8)=0$

$k(k-2)(k^2+2k+4)=0$

$2^x = 0$ of $2^x = 2$ of $\left(2^x\right)^2 + 2(2^x)+4=0$

$\therefore x=1$

Alternatief:

$2^{4x} - 8.2^x = 0$

$2^{4x} = 8.2^x$

$\dfrac{2^{4x}}{2^x} = 8$

$2^{3x} = 8$

$2^{3x} = 2^3$

$\therefore x=1$

(3) $3^x\left(x^2-3x+2\right) \le 0$

$3^x > 0$ dus is 0 nie 'n kritieke waarde nie

Kritieke waardes: 1 ; 2

$1 \le x \le 2$

(d) $9x^2 - 12px + 4p^2 = 0$

$\Delta = \left(-12p\right)^2 - 4(9)(4p^2)$

$\Delta = 144p^2 - 144p^2$

Vir gelyke wortels: $\Delta = 0$

$\therefore 0=0$, dus sal alle reële waardes van p 'n volkome vierkant en gelyke wortels tot gevolg hê.

Alternatief:

$9x^2 - 12px + 4p^2 = 0$

$(3x-2p)(3x-2p)=0$

Alle reële waardes van p sal 'n volkome vierkant en gelyke wortels tot gevolg hê.

VRAAG 10

(a) $f'(x) = -3x^2 + 2bx + c$

$f'(-x) = -3(-x)^2 + 2b(-x) + c$

$f'(-x) = -3x^2 - 2bx + c$

Vir: $\boldsymbol{f'(x) = f'(-x)}$

$-3x^2 + 2bx + c = -3x^2 - 2bx + c$

$-4bx = 0 \quad \therefore \quad b = 0$

$g(x) = 2x + d$

$g'(x) = 2$

$g''(x) = 0$

$f'(-1) = -3(-1)^2 + 2b(-1) + c$

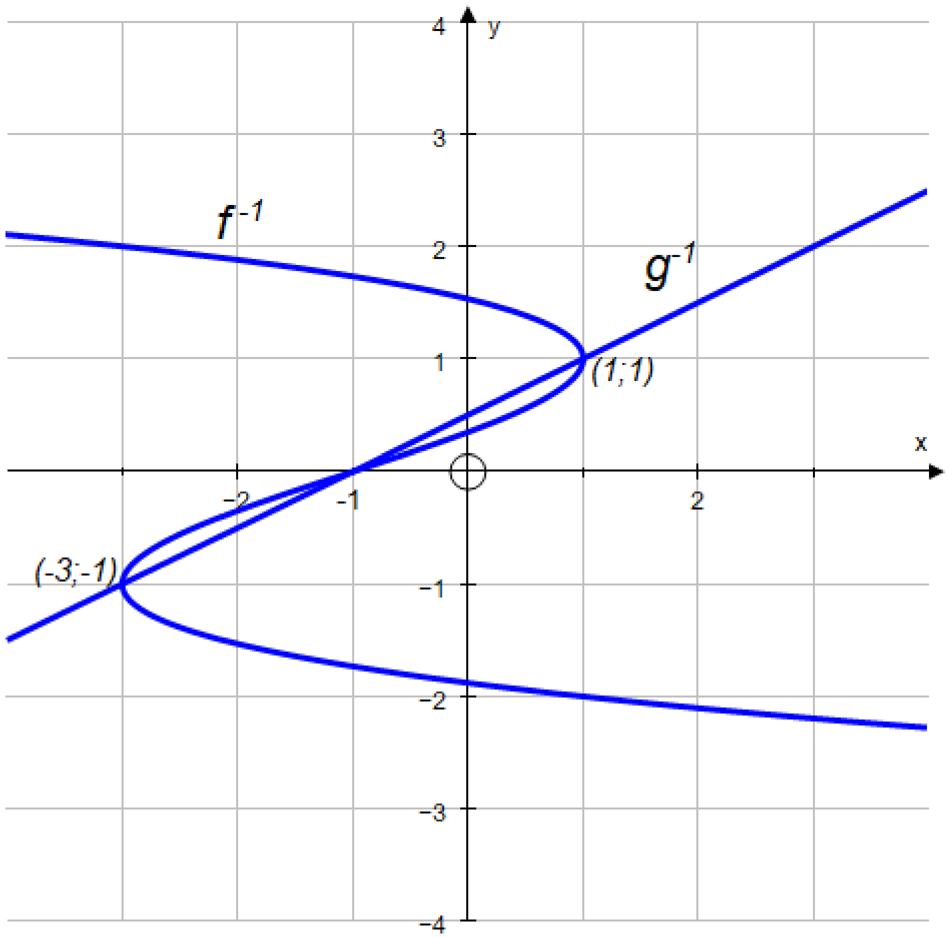
$f'(-1) = -3 - 2b + c$

Uit: $f'(-1) = g''(2)$

$-3 - 2b + c = 0$ vervang $b = 0$

$c = 3$

(b)



VRAAG 11

$(AB)^2 = \left(10\sqrt{5}\right)^2 - \left[\left(475 - 10x - x^2\right)^{\frac{1}{2}}\right]^2$ … Pythagoras

$(AB)^2 = 500 - 475 + 10x + x^2$

$AB = \sqrt{25 + 10x + x^2}$

$AB = \sqrt{(5 + x)^2}$

$AB = 5 + x$

Volume van die keël $= \dfrac{1}{3}\pi\left[\left(475 - 10x - x^2\right)^{\frac{1}{2}}\right]^2 (x + 5)$

Volume van die keël $= \dfrac{1}{3}\pi(475 - 10x - x^2)(x + 5)$

Volume van die keël $= \dfrac{1}{3}\pi\left(475x - 10x^2 - x^3 + 2375 - 50x - 5x^2\right)$

Volume van die keël $= \dfrac{1}{3}\pi\left(-x^3 - 15x^2 + 425x + 2375\right)$

Volume van die keël $= -\dfrac{1}{3}\pi x^3 - 5\pi x^2 + \dfrac{425}{3}\pi x + \dfrac{2375}{3}\pi$

$\dfrac{dV}{dx} = -\pi x^2 - 10\pi x + \dfrac{425}{3}\pi$

Vir maksimum:

$-\pi\left(x^2 + 10x - \dfrac{425}{3}\right) = 0$

$x \approx 7{,}9$ of $x \approx -17{,}9$

Volume van die keël $= -\dfrac{1}{3}\pi(7{,}9)^3 - 5\pi(7{,}9)^2 + \dfrac{425}{3}\pi(7{,}9) + \dfrac{2375}{3}\pi$

Volume van die keël $\approx 4506{,}42$ cm^3

VRAAG 12

(a) Vergelyking van die lyn: $y = mx + 1$

$m = \frac{-1-1}{1-0} = -2$

Vir *x*-afsnit, laat y = 0

$0 = -2x + 1$

$x = \frac{1}{2}$

Negatiewe gradiënt van grafiek van tweede afgeleide, dus is die kromme konkaaf afwaarts vir: $x > \frac{1}{2}$

(b)

Totaal: 150 punte